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A Note on Slavnov-Taylor Identities in the Causal Epstein-Glaser Approach

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Abstract. - An alternative approach to perturbative Yang-Mills theories in 3+1 dimensional space-time based on the causal Epstein-Glaser method in QFT was recently proposed.
In this short note we show that the set of identities between C-number distributions expressing nonabelian gauge invariance in the causal approach imply identities which are analogous to the well-known Slavnov-Taylor identities. We explicitly derive the Z-factor relations at one-loop level.

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Nonabelian Gauge Invariance was recently analysed in the causal Epstein-Glaser approach to perturbative QFT [1]. In this approach the S-matrix is directly constructed in the Fock space of the free asymptotic fields in the form of a formal power series

$$S(g) = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int d^4x_1 ... d^4x_n \, T_n(x_1, ..., x_n) g(x_1) ... g(x_n), \tag{1}$$

where $g(x)$ is a tempered test function which switches the interaction. The central objects are the n-point distributions T_n which may be viewed as mathematically well-defined time-ordered products. The defining equations of the theory in the causal formalism are the fundamental (anti-)commutation relations of the free field operators, their dynamical equations and the specific coupling of the theory $T_{n=1}$. The n-point distributions T_n in (1) are then constructed inductively from the given first order $T_{n=1}$ according to the Epstein-Glaser construction [2] which allows a direct construction of the renormalized (finite) perturbation series without any intermediate modifications. The physical infrared problem is naturally separated by the adiabatic switching of the S-matrix $S(g)$ with a tempered test function g.
Considering Yang-Mills theory in four space-time dimensions, the corresponding specific coupling in the Feynman gauge is

$$\begin{aligned} T_1 = igf_{abc}(\frac{1}{2} : A_{\mu a} A_{\nu b} F_c^{\nu\mu} : - : A_{\mu a} u_b \partial^\mu \tilde{u}_c : + \\ + \alpha \quad \partial_\mu (: u_a \tilde{u}_b A_c^\mu :) - \beta : \partial_\mu A_a^\mu \tilde{u}_b u_c :) \end{aligned} \tag{2}$$

where α and β are free constants. All field operators herein are well-defined free fields and these are the only quantities appearing in the whole theory. The double dots denote their normal ordering. The specific coupling $T_{n=1}(x)$ of the theory does not contain quadrilinear terms proportional to g^2, the four-gluon-vertex nor the four-ghost-vertex. Both terms are automatically generated in second order by our gauge invariance condition [3].
$A_{\mu a}(x)$ are the (free) gauge potentials satisfying the commutation relations (Feynman gauge)

$$[A_a^{(-)\mu}(x), A_b^{(+)\nu}(y)] = i\delta_{ab} g^{\mu\nu} D_0^{(+)}(x-y), \tag{3}$$

where $A^{(\pm)}$ are the emission and absorption parts of A and $D_0^{(\pm)}$ the (mass zero) Pauli-Jordan distributions. $u_a(x)$ and $\tilde{u}_a(x)$ are the free massless fermionic ghost fields fulfilling the anticommutation relations

$$\{u_a^{(\pm)}(x), \tilde{u}_b^{(\mp)}(y)\} = -i\delta_{ab} D_0^{(\mp)}(x-y). \tag{4}$$

f_{abc} denotes the usual antisymmetric structure constants of the gauge group $SU(N)$. The time-dependence of A, u and $\tilde{u}$ in Feynman gauge is given by the wave equation

$$\square A_a^\mu(x) = 0, \quad \square u_a(x) = 0, \quad \square \tilde{u}_a(x) = 0, \tag{5}$$

We define

$$F_a^{\mu\nu} \stackrel{\text{def}}{=} \partial^\mu A_a^\nu - \partial^\nu A_a^\mu. \tag{6}$$

Now one considers the linear (abelian!) BRS transformations of the free asymptotic field operators. The generator of the abelian operator transformations is the charge

$$Q \stackrel{\text{def}}{=} \int d^3x \, (\partial_\nu A_a^\nu \overleftrightarrow{\partial}_0 u_a), \quad Q^2 = 0, \tag{7}$$

with the (anti-)commutation relations

$$[Q, A_\mu^a]_- = i\partial_\mu u_a, \quad \{Q, \tilde{u}_a\}_+ = -i\partial_\nu A_a^\nu, \quad \{Q, u_a\}_+ = 0, \quad [Q, F_{\mu\nu}^a]_- = 0. \tag{8}$$

In addition to the charge Q, one defines the ghost charge

$$Q_c := i \int d^3x : (\tilde{u} \overleftrightarrow{\partial}_0 u) : \tag{9}$$

In the algebra, generated by the fundamental field operators, one introduces a gradation by the ghost number $G(\hat{A})$ which is given on the homogenous elements by

$$[Q_c, \hat{A}] = -G(\hat{A}) \cdot \hat{A}. \tag{10}$$

One can define an anti-derivation d_Q in the graded algebra by

$$d_Q \hat{A} := Q\hat{A} - (e^{i\pi Q_c} \hat{A} e^{-i\pi Q_c})Q \tag{11}$$

The anti-derivation d_Q is obviously homogenous of degree (-1) and satisfies $d_Q^2 = 0$.

Nonabelian gauge invariance in the causal approach means that the commutator of the specific coupling (2) with the charge Q is a divergence (in the sense of vector analysis):

$$d_Q T_{n=1} = i\partial_\nu[igf_{abc}(: A_\nu^a u_b F_c^{\nu\mu} : -\tfrac{1}{2} : u_a u_b \partial^\nu \tilde{u}_c : -\alpha i d_Q(: u_a \tilde{u}_b A_c^\nu :))] \stackrel{\text{def}}{=} i\partial_\nu T_{1/1}^\nu \tag{12}$$

The second term in (2) (the gluon-ghost-coupling) is essential that $d_Q T_{n=1}$ can be written as a divergence. Note the different compensation of terms in the invariance equation (2) compared with the invariance of the Yang-Mills Lagrangean under the full BRS-transformations of the interaction fields in the conventional formalism.
$T_{n=1}$ in (1) represents the most general gauge invariant (in the sense of (12)) and Lorentz invariant operator, which is also invariant in regard to the global $SU(n)$ group and in regard to the discrete symmetry transformations C, P, T , and which has maximal mass dimension four and ghost charge zero. Note that terms with four operators are ruled out by the gauge invariance condition (12). As we have already mentioned above, the four-gluon- and four-ghost-couplings are automatically generated by the gauge invariance condition in the second order. Moreover, we left out all possible gauge invariant terms with two operators because the information of quadratic terms are already contained in the fundamental (anti-)commutation relations and the dynamical equations of the operators.
$T_{n=1}$ in (2) is also anti-gauge invariant in the sense that

$$[\bar{Q}, T_{n=1}] \quad (\text{where} \quad \bar{Q} := \int d^3x \, (\partial_\nu A_a^\nu \overleftrightarrow{\partial}_0 \tilde{u}_a) \quad \text{with} \quad \bar{Q}^2 = 0)$$

is also a divergence (in the sense of vector analysis).

The condition of nonabelian operator gauge invariance in the causal approach is expressed in every order of perturbation theory separately by a simple commutator relation of the n-point distributions T_n with the charge Q, the generator of the free operator gauge transformations:

$$[Q, T_n(x_1, ..., x_n)] = d_Q T_n(x_1, ..., x_n) = i \sum_{l=1}^{n} \partial_\mu^{x_l} T_{n/l}^\mu(x_1, ..., x_n), \tag{13}$$

where $T^{\nu}_{n/l}(x_1, \ldots, x_n)$ are n-point distributions of an extended theory which contains, in addition to the usual Yang-Mills couplings $T_{n=1}(x)$ (2), the so-called Q-vertex $T^{\nu}_{1/1}(x)$ which already occurs in (11) as a divergence-representation of $[Q, T_1]$. The first order S-matrix of the extended theory is equal to

$$S_1(g_0, g_1) \stackrel{\text{def}}{=} \int d^4x \, [T_1(x) g_0(x) + T^{\nu}_{1/1}(x) g_{1\nu}(x)]. \tag{14}$$

and $g_1 = (g_{1\nu})_{\nu=0,1,2,3} \in (\mathcal{S}(\mathbf{R}^4))^4$ must be an anti-commuting C-number field. The higher orders are determined by the usual inductive Epstein Glaser construction up to local normalization terms. The $T^{\mu}_{n/l}$ are the n-point distributions of the extended theory with one Q-vertex at x_l, all other $n-1$ vertices are ordinary Yang-Mills vertices (2) (for details see [1]).
The representation of $[Q, T_{n=1}]$ as a divergence is in general not unique. The most general Q-vertex $\tilde{T}^{\nu}_{1/1}$ with the same mass dimension and ghost number as $T^{\nu}_{1/1}$ in (12) is the following:

$$[Q, T_1] = i\partial_\nu [T^{\nu}_{1/1} + \gamma B^{\nu}_{1/1}] \stackrel{\text{def}}{=} i\partial_\nu \tilde{T}^{\nu}_{1/1}$$

$$\text{with} \qquad B^{\nu}_{1/1} = igf_{abc}\partial_\mu (: u_a A^{\mu}_b A^{\nu}_c :), \quad \partial_\nu B^{\nu}_{1/1} = 0, \quad \gamma \in \mathrm{C} \quad \text{free.} \tag{15}$$

The choice of γ has just practical reasons and has no physical consequences.

We claimed in [1] that the simple operator condition (13) involving only well-defined asymptotic field operators expresses the full content of the nonabelian gauge structure of the quantized theory. We proved this condition by induction on the order n of perturbation theory following the causal construction of T_n and $T^{\nu}_{n/l}$. We also proved that this condition implies the unitarity of the S-matrix in the physical subspace , i.e. the decoupling of the unphysical degrees of freedom in the theory. Thus, the concept of abelian gauge transformations of the free field operators is sufficient in order to derive the most important consequence of nonabelian gauge invariance in perturbative quantum field theory.
However, one may doubt if this simple operator equation represents the whole content of nonabelian gauge invariance in perturbation theory, for example the consequences for (amputated) Greensfunctions, namely the Slavnov-Taylor identities. The purpose of this note is to show that the equation (13) also contains this latter information.
In [1] we expressed the operator gauge invariance condition by a set of identities between C-number distributions. These C-number identities for gauge invariance (so-called cg-identities) are sufficient for the operator condition (13). We have rewritten these identities in the appendix. Note that they correspond to the specific choice $\alpha = 0$ and $\beta = 0$ in (2), which corresponds to the Faddeev Popov specific coupling. Moreover, we have chosen $\gamma = 0$, the most suitable choice for the Q-vertex.
It is an advantage of the causal approach that the physical infrared problem is naturally separated by adiabatic switching of the S-matrix by a tempered testfunction g and also absent before the limit $g \to 1$ is taken. So all examinations regarding gauge invariance and unitarity are mathematically well-defined. But one has to pay a prize: In order to express the operator gauge invariance condition (13) in a set of identities between C-number distributions, we have to work out the explicit form of the divergence in (12) and (13)([1],[4]). Moreover, one has to distinguish the operator and its derivative , which implies the relative largeness of the set of cg-identities.

However, we can derive 5 summed identities from this large set of identities which are totally analogous to the Slavnov-Taylor identities.
In fact, we can eliminate all distributions with one Q-vertex besides the divergences in regard to the inner variables. One arrives at relations which almost only involve distributions of the orginal theory: In order to get the summed 2-leg identity, one has to insert (A.1) into (A.2). Besides the 3-leg identity (A.3) one attends another summed (3-leg) identity by inserting (A.6) into (A.5), then (A.5) into (A.4). The 4-leg identities are treated analogously: Inserting (A.12) into (A.8), we get the first summed 4-leg identity and inserting (A.11) into (A.10), then (A.10) into (A.9) and finally (A.9) into (A.7), we arrive at the second summed 4-leg identity:

• In the first step we define the following summed distributions. These definitions are natural, because the defined distributions represent in each case the sum of all distributions which would contribute to the same operator in the adiabatic limit (Partial integrating is formally possible in the adiabatic limit.). The crucial point is that also the four-gluon terms proportional to δ which originate from the induced four-gluon normalisation term in second order ([1]) contribute to the operator where all external legs are attached to different vertices and therefore have to be included in the definitions:

$$\begin{gathered}\Pi^{\kappa\nu}_{AA}(x_1,x_2,\ldots):=t^{\kappa\nu}_{AA}(x_1,x_2,\ldots)+\\ -2\partial^{x_2}_\lambda t^{\kappa\lambda\nu}_{AF}(x_1,x_2,\ldots)-2\partial^{x_1}_\lambda t^{\lambda\kappa\nu}_{FA}(x_1,x_2,\ldots)+4\partial^{x_1}_\lambda\partial^{x_2}_\tau t^{\lambda\kappa\tau\nu}_{FF}(x_1,x_2,\ldots)\end{gathered} \tag{16}$$

$$\Pi^{\kappa l\nu}_{uA}(x_1,x_2,\ldots):=t^{\kappa l\nu}_{uA}(x_1,x_2,\ldots)-2\partial^{x_2}_\lambda t^{\kappa l\lambda\nu}_{uF}(x_1,x_2,\ldots),l>2$$

$$\Pi^{\mu\nu}_{u\overline{u}A}(x_1,x_2,x_3,\ldots):=t^{\mu\nu}_{u\overline{u}A}(x_1,x_2,x_3,\ldots)-2\partial^{x_3}_\kappa t^{\mu\kappa\nu}_{u\overline{u}F}(x_1,x_2,x_3,\ldots)$$

$$\begin{gathered}\Pi^{\alpha\mu\nu}_{AAA}(x_1,x_2,x_3,\ldots):=t^{\alpha\mu\nu}_{AAA}(x_1,x_2,x_3,\ldots)+\\ +2g\delta(x_1-x_2)t^{\nu\alpha\mu}_{AF}(x_3,x_2,\ldots)-2g\delta(x_1-x_3)t^{\mu\alpha\nu}_{AF}(x_2,x_3,\ldots)+2g\delta(x_2-x_3)t^{\alpha\mu\nu}_{AF}(x_1,x_2,\ldots)+\\ -2\partial^{x_1}_\kappa[t^{\kappa\alpha\mu\nu}_{FAA}(x_1,x_2,x_3,\ldots)+2g\delta(x_2-x_3)t^{\mu\nu\kappa\alpha}_{FF}(x_3,x_1,\ldots)]+\\ -2\partial^{x_2}_\kappa[t^{\alpha\kappa\mu\nu}_{AFA}(x_1,x_2,x_3,\ldots)-2g\delta(x_1-x_3)t^{\alpha\nu\kappa\mu}_{FF}(x_3,x_2,\ldots)]+\\ -2\partial^{x_3}_\kappa[t^{\alpha\mu\kappa\nu}_{AAF}(x_1,x_2,x_3,\ldots)+2g\delta(x_1-x_2)t^{\kappa\nu\alpha\mu}_{FF}(x_3,x_2,\ldots)]+\\ +4\partial^{x_2}_\kappa\partial^{x_3}_\lambda t^{\alpha\kappa\mu\lambda\nu}_{AFF}(x_1,x_2,x_3,\ldots)+4\partial^{x_1}_\kappa\partial^{x_2}_\lambda t^{\kappa\alpha\lambda\mu\nu}_{FFA}(x_1,x_2,x_3,\ldots)-8\partial^{x_1}_\kappa\partial^{x_2}_\lambda\partial^{x_3}_\sigma t^{\kappa\alpha\lambda\mu\sigma\nu}_{FFF}(x_1,x_2,x_3,...)\end{gathered}$$

$$\begin{gathered}\Pi^{\alpha l\mu\nu}_{uAA}(x_1,x_2,x_3,\ldots):=t^{\alpha l\mu\nu}_{uAA}(x_1,x_2,x_3,\ldots)+\\ -2\partial^{x_3}_\kappa t^{\alpha l\mu\kappa\nu}_{uAF}(x_1,x_2,x_3,\ldots)-2\partial^{x_2}_\kappa t^{\alpha l\kappa\mu\nu}_{uFA}(x_1,x_2,x_3,\ldots)+4\partial^{x_2}_\kappa\partial^{x_3}_\lambda t^{\alpha l\kappa\mu\lambda\nu}_{uFF}(x_1,x_2,x_3,\ldots)+\\ +2g\delta(x_2-x_3)t^{\alpha(l-1)\mu\nu}_{uF}(x_1,x_2,\ldots),l>3\end{gathered}$$

$$\Pi^{\alpha\mu}_{Au\overline{u}}(x_1,x_2,x_3,.....) := t^{\alpha\mu}_{Au\overline{u}}(x_1,x_2,x_3,....) - 2\partial^{x_1}_\kappa t^{\kappa\alpha\mu}_{Fu\overline{u}}(x_1,x_2,x_3,...)$$

$$\begin{aligned}\Pi^{\alpha\mu\nu}_{uA\overline{u}Aabcd}&(x_1,x_2,x_3,x_4,...) := t^{\alpha\mu\nu}_{uA\overline{u}Aabcd}(x_1,x_2,x_3,x_4,....)+\\ &-2\partial^{x_4}_\kappa t^{\alpha\mu\kappa\nu}_{uA\overline{u}Fabcd}(x_1,x_2,x_3,x_4,....) - 2\partial^{x_2}_\kappa t^{\kappa\alpha\mu\nu}_{uF\overline{u}Aabcd}(x_1,x_2,x_3,x_4,...)+\\ &+4\partial^{x_2}_\kappa\partial^{x_4}_\lambda t^{\kappa\alpha\mu\lambda\nu}_{uF\overline{u}Fabcd}(x_1,x_2,x_3,x_4,....) + 2gf_{bdr}f_{acr}\delta(2-4)t^{\mu\alpha\nu}_{u\overline{u}F}(x_1,x_3,x_4,.....)\end{aligned}$$

$$\bar{\Pi}^{3\nu}_{uu\overline{u}Aabcd}(x_1,x_2,x_3,x_4,...) := \bar{t}^{3\nu}_{uu\overline{u}Aabcd}(x_1,x_2,x_3,x_4,...) - 2\partial^{x_4}_\alpha \bar{t}^{3\alpha\nu}_{uu\overline{u}Fabcd}(x_1,x_2,x_3,x_4,...)$$

$$\Pi^{\alpha l\mu\nu}_{uu\overline{u}Aabcd}(x_1,x_2,x_3,x_4,...) := t^{\alpha l\mu\nu}_{uu\overline{u}Aabcd}(x_1,x_2,x_3,x_4,...) - 2\partial^{x_4}_\kappa t^{\alpha l\mu\kappa\nu}_{uu\overline{u}Fabcd}(x_1,x_2,x_3,x_4,...), l > 4$$

Analogously, one can define $\Pi^{\alpha\nu\kappa\lambda}_{AAAAabcd}$ and $\Pi^{\alpha\kappa\lambda}_{u\overline{u}AAabcd}$.

• Having defined these summed distributions we arrive at the summed two-leg identity by inserting (A.2) into (A.1) and using the new definitions.

$$\begin{aligned}\partial^{x_1}_\kappa\Pi^{\kappa\nu}_{AA}(x_1,x_2,x_3,\ldots,x_{n-1}) - \partial^\alpha_{x_2}[\partial^\alpha_{x_2}t^\nu_{u\overline{u}}(x_1,x_2,x_3,\ldots,x_{n-1}) - (\alpha\leftrightarrow\nu)]+\\ +\sum_{l=3}^n \partial^{x_l}_\kappa\Pi^{\kappa l\nu}_{uA}(x_1,x_2,x_3,\ldots,x_{n-1}) = 0\end{aligned} \tag{17}$$

Inserting (A.6) and (A.5) into (A.4) and using the new definitions, we arrive at the first summed three-leg identities of gauge invariance:

$$\begin{aligned}&\partial^{x_1}_\alpha\Pi^{\alpha\mu\nu}_{AAA}(x_1,x_2,x_3,x_4,\ldots,x_n)+\\ &+[\big(\partial^{x_2}_\alpha[\partial^\alpha_{x_2}\Pi^{\mu\nu}_{u\overline{u}A}(x_1,x_2,x_3,x_4,\ldots,x_n) - (\alpha\leftrightarrow\mu)]\big) - \big((x_2,\nu)\longleftrightarrow(x_3,\mu)\big)]+\\ &+g[\delta(x_1-x_2)-\delta(x_1-x_3)]\Pi^{\mu\nu}_{AA}(x_2,x_3,x_4,\ldots,x_n)+\\ &+g[\big(\partial^{x_2}_\alpha[\delta(x_2-x_3)g^{\alpha\mu}t^\nu_{u\overline{u}}(x_1,x_2,x_4,\ldots,x_n) - (\alpha\leftrightarrow\nu)]\big) - \big((x_2,\nu)\longleftrightarrow(x_3,\mu)\big)]+\\ &+\sum_{l=4}^n \partial^l_\alpha\Pi^{\alpha l\mu\nu}_{uAA}(x_1,x_2,x_3,x_4,\ldots,x_n) = 0\end{aligned} \tag{18}$$

We can rewrite equation (A.3) as the second summed three-leg identity:

$$\begin{aligned}&\partial^{x_1}_\alpha\Pi^{\alpha\mu}_{Au\tilde{u}}(x_1,x_2,x_3,\ldots) + \partial^{x_2}_\alpha\Pi^{\alpha\mu}_{Au\tilde{u}}(x_2,x_1,x_3,\ldots) + \partial^\mu_{x_3}\vec{t}^{3}_{uu\tilde{u}}(x_1,x_2,x_3,\ldots)+\\ &+\sum_{l=4}^n \partial^{x_l}_\alpha t^{\alpha l\mu}_{uu\tilde{u}}(x_1,x_2,x_3,\ldots) + g\delta(x_1-x_2)t^\mu_{u\tilde{u}}(x_2,x_3,\ldots)+\\ &-g\delta(x_1-x_3)t^\mu_{u\tilde{u}}(x_2,x_3,\ldots) - g\delta(x_2-x_3)t^\mu_{u\tilde{u}}(x_1,x_3,\ldots) = 0.\end{aligned} \tag{19}$$

Inserting (A.12) into (A.8), we get the first summed four-leg identity:

$$0=-\Big[\partial_\alpha^{x_2}\Pi^{\alpha\mu\nu}_{uA\tilde{u}Aabcd}(x_1,x_2,x_3,x_4,\ldots)-\Big((a,x_1)\longleftrightarrow(b,x_2)\Big)\Big]+$$

$$+\partial^\mu_{x_3}\bar{\Pi}^{3\nu}_{uu\tilde{u}Aabcd}(x_1,x_2,x_3,x_4,\ldots)-\partial_\alpha^{x_4}\Big[\partial_\nu^{x_4}t^{\mu\alpha}_{uu\tilde{u}\tilde{u}abcd}(x_1,x_2,x_3,x_4,\ldots)-(\nu\leftrightarrow\alpha)\Big]+$$

$$+\sum_{l=5}^{n}\partial_\alpha^{x_l}\Pi^{\alpha l\mu\nu}_{uu\tilde{u}Aabcd}(x_1,x_2,x_3,x_4,\ldots)+$$

$$\begin{aligned}&+g\{f_{abr}f_{cdr}[\delta(x_1-x_2)\Pi^{\mu\nu}_{u\tilde{u}A}(x_2,x_3,x_4\ldots)+\\&-g\{f_{acr}f_{bdr}\delta(x_1-x_3)\Pi^{\mu\nu}_{u\tilde{u}A}(x_2,x_3,x_4,\ldots)]-(a,x_1)\longleftrightarrow(b,x_2)\}+\\&+g\{f_{adr}f_{bcr}\delta(x_1-x_4)\Pi^{\mu\nu}_{u\tilde{u}A}(x_2,x_3,x_4\ldots)-(a,x_1)\longleftrightarrow(b,x_2)\}+\\&+gf_{abr}f_{cdr}\delta(x_3-x_4)g^{\nu\mu}\bar{t}^3_{uu\tilde{u}}(x_1,x_2,x_4,\ldots)]\end{aligned}\tag{20}$$

Inserting (A.11) into (A.10), then (A.10) into (A.9) and finally (A.9) into (A.7), we arrive at the second summed 4-leg identity :

$$\partial_\alpha^{x_1}\Pi^{\alpha\nu\kappa\lambda}_{AAAAabcd}(1,2,3,4,5,\ldots)+\Big\{(-\partial_\alpha^{x_2}\partial^\nu_{x_2}\Pi^{\alpha\kappa\lambda}_{u\overline{u}AAabcd}(1,2,3,4,5,\ldots))-((\alpha\leftrightarrow\nu)\Big\}+$$

$$+\Big\{(b,\nu,x_2)\to(c,\kappa,x_3)\to(d,\lambda,x_4)\to(b,\nu,x_2)\Big\}+\quad degenerate\quad terms=0\tag{21}$$

These 5 (summed) identities are alike the Slavnov-Taylor identities. They can directly compared with explicit identities which can be found in the literature [5]. But note that the summed cg-identities above are more general because in the usual identities the adiabatic limit $g\to 1$ in the inner variables is already taken.

Finally, we derive the well-known relation between the Z-factors of the gluon vertex, the gluon propagator, the ghost vertex and the ghost propagator at one-loop level [5] from these summed Cg-identities :

• One easily checks that the following (local) renormalisations of the self energy distributions are compatible with the first summed identity of gauge invariance (17) (and also with Lorentz invariance, all the discrete symmetries and pseudo-unitarity) in the nth step of the inductive construction. Because we are interested in the comparison with the Slavnov-Taylor identities, we state only the relevant local normalisation terms which survive in the adiabatic limit in regard to the inner coordinates:

$$\begin{aligned}&\Pi^{\mu\nu}_{AA}+C^{n-1}_{AA}[\partial^\mu_{x_1}\partial^\nu_{x_1}-g^{\mu\nu}\partial_{x_1}\partial_{x_1}]\delta^{n-2}\\&t^\nu_{u\overline{u}}+C^{n-1}_{u\overline{u}}\partial^\nu_{x_2}\delta^{n-2}\end{aligned}\tag{22}$$

•The possible renormalisations of the two vertices (compatible with Lorentz invariance, discrete symmetries and pseudo-unitarity) are the following:

$$\begin{aligned}&\Pi^{\alpha\mu\nu}_{AAA}+C^n_{AAA}[g^{\alpha\mu}(\partial^\nu_{x_1}-\partial^\nu_{x_2})+g^{\alpha\nu}(\partial^\mu_{x_3}-\partial^\mu_{x_1})+g^{\mu\nu}(\partial^\alpha_{x_2}-\partial^\alpha_{x_3})]\delta^{n-1}\\&\Pi^{\mu\nu}_{u\overline{u}A}+C^n_{u\overline{u}A}\delta^{n-1}g^{\mu\nu}\end{aligned}\tag{23}$$

The second summed identity (18) implies the following relation between these four normalisation constants in the nth step of the inductive construction:

$$gC_{AA}^{n-1} + C_{u\overline{u}A}^{n} - gC_{u\overline{u}}^{n-1} - C_{AAA}^{n} = 0 \tag{24}$$

Because of $Z_i := 1 + C_i$ (Note our conventions in the ghost sector!), we directly get the well-known relation between the Z-factors at one-loop level:

$$\frac{Z_{AA}}{Z_{AAA}} = \frac{Z_{u\overline{u}}}{Z_{u\overline{u}A}} \tag{25}$$

The interpretation of this relation is slightly different in the causal approach: It represents the restrictions by gauge invariance on finite normalisation terms only. We do not need any infinite part in the Z-factors to absorb divergences in the causal approach.
Using the two summed 4-leg identities or the identities, one can analogously deduce the corresponding relation of the Z-factor of the four-gluon vertex.

Appendix Cg-Identities

The conventions of denoting operator-valued distributions are as in [1]:

$$t^{\alpha 2}_{AB\ldots ab\ldots}(x_1, x_2, \ldots) : A^a(x_1) B^b(x_2) \ldots :$$

means an operator-valued distribution with external field operators (legs) A^a and B^b, a and b are colour indices. The subscripts $\alpha 2$ show that this term belongs to $T^{\alpha}_{n/2}(x_1, x_2, \ldots)$ with Q-vertex at the second argument of the numerical distribution t. All 2-leg distributions contain the colour tensor δ_{ab}, all 3-leg distributions the colour factor f_{abc}. Therefore we define the numerical distributions without these colour factors .

For : $\Omega := \delta_{ab} : u_a(x_1) A^b_\nu(x_2) :$ we obtain

$$\partial^1_\alpha t^{\alpha\nu}_{AA} + \frac{1}{2}\partial^2_\alpha [t^{\alpha 2\nu}_{uA} - t^{\nu 2\alpha}_{uA}] + \sum_{l=3}^{n} \partial^l_\alpha t^{\alpha l \nu}_{uA} = 0, \tag{A.1}$$

For : $\Omega := \delta_{ab} : u_a(x_1) F^b_{\mu\nu}(x_2) :$

$$\partial^1_\alpha t^{\alpha\mu\nu}_{AF} + \frac{1}{2}[\partial^\mu_2 t^\nu_{u\tilde{u}} - \partial^\nu_2 t^\mu_{u\tilde{u}}] + \frac{1}{4}[t^{\mu 2\nu}_{uA} - t^{\nu 2\mu}_{uA}] + \sum_{l=3}^{n} \partial^l_\alpha t^{\alpha l \mu\nu}_{uF} = 0. \tag{A.2}$$

For : $\Omega := f_{abc} : u^a(x_1) u^b(x_2) \partial_\mu \tilde{u}^c(x_3) :$

$$0 = \partial^{x_1}_\alpha t^{\alpha\mu}_{Au\tilde{u}}(x_1, x_2, x_3, \ldots) + \partial^{x_2}_\alpha t^{\alpha\mu}_{Au\tilde{u}}(x_2, x_1, x_3, \ldots) + \partial^\mu_{x_3} \vec{t}^{\,3}_{uu\tilde{u}}(x_1, x_2, x_3, \ldots)$$

$$+\sum_{l=4}^{n}\partial_\alpha^{x_l}t^{\alpha l\mu}_{uu\tilde{u}}(x_1,x_2,x_3,\ldots)+g\delta(x_1-x_2)t^{\mu}_{u\tilde{u}}(x_2,x_3,\ldots)$$

$$-g\delta(x_1-x_3)t^{\mu}_{u\tilde{u}}(x_2,x_3,\ldots)-g\delta(x_2-x_3)t^{\mu}_{u\tilde{u}}(x_1,x_3,\ldots). \qquad (A.3)$$

For : $\Omega := f_{abc} : u^a(x_1)A^b_\mu(x_2)A^c_\nu(x_3) :$

$$0=\partial_\alpha^{x_1}t^{\alpha\mu\nu}_{AAA}(x_1,x_2,x_3,x_4,\ldots)-\frac{1}{2}\partial_\alpha^{x_2}\Big[t^{\alpha 3\nu\mu}_{uAA}(x_1,x_3,x_2,x_4,\ldots)-(\alpha\leftrightarrow\mu)\Big]$$

$$+\frac{1}{2}\partial_\alpha^{x_3}\Big[t^{\alpha 3\mu\nu}_{uAA}(x_1,x_2,x_3,x_4,\ldots)-(\alpha\leftrightarrow\nu)\Big]+\sum_{l=4}^{n}\partial_\alpha^{x_l}t^{\alpha l\mu\nu}_{uAA}(x_1,x_2,x_3,x_4,\ldots)$$

$$+g[\delta(x_1-x_2)-\delta(x_1-x_3)]t^{\mu\nu}_{AA}(x_2,x_3,x_4,\ldots)$$

$$-g\delta(x_2-x_3)\frac{1}{2}\Big[t^{\mu 2\nu}_{uA}(x_1,x_2,x_4,\ldots)-(\mu\leftrightarrow\nu)\Big]. \qquad (A.4)$$

For : $\Omega := f_{abc} : u^a(x_1)A^b_\mu(x_2)F^c_{\nu\lambda}(x_3) :$

$$0=\frac{1}{4}\Big[t^{\nu 3\mu\lambda}_{uAA}(x_1,x_2,x_3,\ldots)-(\nu\leftrightarrow\lambda)\Big]$$

$$+\partial_\alpha^{x_1}t^{\alpha\mu\nu\lambda}_{AAF}(x_1,x_2,x_3,\ldots)+\frac{1}{2}\partial_\alpha^{x_2}\Big[t^{\alpha 2\mu\nu\lambda}_{uAF}(x_1,x_2,x_3,\ldots)-(\alpha\leftrightarrow\mu)\Big]$$

$$+\frac{1}{2}\Big[\partial^\nu_{x_3}t^{\lambda\mu}_{u\tilde{u}A}(x_1,x_3,x_2,\ldots)-(\nu\leftrightarrow\lambda)\Big]+\sum_{l=4}^{n}\partial_\alpha^{x_l}t^{\alpha l\mu\nu\lambda}_{uAF}(x_1,x_2,x_3,\ldots)$$

$$+g[\delta(x_1-x_2)-\delta(x_1-x_3)]t^{\mu\nu\lambda}_{AF}(x_2,x_3,x_4,\ldots)$$

$$+\frac{g}{2}\Big[g^{\mu\nu}\delta(x_2-x_3)t^{\lambda}_{u\tilde{u}}(x_1,x_3,x_4,\ldots)-(\nu\leftrightarrow\lambda)\Big]. \qquad (A.5)$$

For : $\Omega := f_{abc} : u^a(x_1)F^b_{\mu\tau}(x_2)F^c_{\nu\lambda}(x_3) :$

$$0=\frac{1}{4}\Big[t^{\mu 2\tau\nu\lambda}_{uAF}(x_1,x_2,x_3\ldots)-(\mu\leftrightarrow\tau)\Big]-\frac{1}{4}\Big[t^{\nu 2\lambda\mu\tau}_{uAF}(x_1,x_3,x_2\ldots)-(\nu\leftrightarrow\lambda)\Big]$$

$$+\partial_\alpha^{x_1}t^{\alpha\mu\tau\nu\lambda}_{AFF}(x_1,x_2,x_3,\ldots)+\frac{1}{2}\Big[\partial^\tau_{x_2}t^{\mu\nu\lambda}_{u\tilde{u}F}(x_1,x_2,x_3\ldots)-(\mu\leftrightarrow\tau)\Big]$$

$$-\frac{1}{2}\Big[\partial^\lambda_{x_3}t^{\nu\mu\tau}_{u\tilde{u}F}(x_1,x_3,x_2\ldots)-(\lambda\leftrightarrow\nu)\Big]$$

$$+\sum_{l=4}^{n}\partial_\alpha^{x_l}t^{\alpha l\mu\tau\nu\lambda}_{uFF}(x_1,x_2,x_3\ldots)$$

$$+g[\delta(x_1-x_2)-\delta(x_1-x_3)]t^{\mu\tau\nu\lambda}_{FF}(x_2,x_3,x_4\ldots). \qquad (A.6)$$

$: \Omega :=: u^a(x_1)A^b_\nu(x_2)A^c_\kappa(x_3)\ A^d_\lambda(x_4) :$

$$0=\partial_\alpha^{x_1}t^{\alpha\nu\kappa\lambda}_{AAAAabcd}(1,2,3,4,5,\ldots)+\tfrac{1}{2}\partial_\alpha^{x_2}[t^{\alpha 2\nu\kappa\lambda}_{uAAAabcd}(1,2,3,4,5,\ldots)-(\alpha\leftrightarrow\nu)]$$

$$+\tfrac{1}{2}\partial_\alpha^{x_3}[t^{\alpha 2\kappa\nu\lambda}_{uAAAacbd}(1,3,2,4,5,\ldots)-(\alpha\leftrightarrow\kappa)]+\tfrac{1}{2}\partial_\alpha^{x_4}[t^{\alpha 2\lambda\nu\kappa}_{uAAAadbc}(1,4,2,3,5,\ldots)-(\alpha\leftrightarrow\lambda)]$$

$$+\sum_{l=5}^{n}\partial_\alpha^{x_l} t^{\alpha l\nu\kappa\lambda}_{uAAAabcd}(1,2,3,4,5,\dots)+$$

$$+\Big\{gf_{abr}f_{cdr}\Big[\delta(1-2)t^{\nu\kappa\lambda}_{AAA}(2,3,4,5,\dots)+\delta(3-4)\tfrac{1}{2}[t^{\kappa 2\lambda\nu}_{uAA}(1,3,2,5,\dots)-(\kappa\leftrightarrow\lambda)]\Big]\Big\}$$

$$+\Big\{(b,\nu,x_2)\to(c,\kappa,x_3)\to(d,\lambda,x_4)\to(b,\nu,x_2)\Big\}, \qquad (A.7)$$

For : $\Omega :=: u^a(x_1)u^b(x_2)\partial_\mu\tilde{u}^c(x_3)A^d_\nu(x_4)$:

$$0=-\Big[\partial_\alpha^{x_2}t^{\alpha\mu\nu}_{uA\tilde{u}Aabcd}(1,2,3,4,5,\dots)-\Big((a,x_1)\longleftrightarrow(b,x_2)\Big)\Big]$$

$$+\partial^\mu_{x_3}\vec{t}^{3\nu}_{uu\tilde{u}Aabcd}(1,2,3,4,5,\dots)+\partial_\alpha^{x_4}\tfrac{1}{2}\Big[t^{\alpha 4\mu\nu}_{uu\tilde{u}Aabcd}(1,2,3,4,5,\dots)-(\alpha\leftrightarrow\nu)\Big]$$

$$+\sum_{l=5}^{n}\partial_\alpha^{x_l}t^{\alpha l\mu\nu}_{uu\tilde{u}Aabcd}(1,2,3,4,5,\dots)$$

$$+g\{f_{adr}f_{bcr}[\delta(1-4)t^{\nu\mu}_{Au\tilde{u}}(4,2,3,5\dots)$$

$$+\delta(2-3)t^{\nu\mu}_{Au\tilde{u}}(4,1,3,5\dots)]\}-g\{(a,x_1)\longleftrightarrow(b,x_2)\}$$

$$+gf_{abr}f_{cdr}[\delta(1-2)t^{\nu\mu}_{Au\tilde{u}}(4,2,3,5\dots)$$

$$+\delta(3-4)g^{\nu\mu}\vec{t}^{3}_{uu\tilde{u}}(1,2,4,5\dots)], \qquad (A.8)$$

For : $\Omega :=: u^a(x_1)F^b_{\kappa\lambda}(x_2)A^c_\mu(x_3)A^d_\nu(x_4)$:

$$0=\partial_\alpha^{x_1}t^{\alpha\kappa\lambda\mu\nu}_{AFAAabcd}(1,2,3,4,5,\dots)+\tfrac{1}{2}[\partial^\lambda_{x_2}t^{\kappa\mu\nu}_{u\tilde{u}AAabcd}(1,2,3,4,5,\dots)-(\lambda\longleftrightarrow\kappa)]$$

$$+\tfrac{1}{2}\partial_\alpha^{x_3}[t^{\alpha 3\kappa\lambda\mu\nu}_{uFAAabcd}(1,2,3,4,5,\dots)-(\alpha\longleftrightarrow\mu)]$$

$$+\tfrac{1}{2}\partial_\alpha^{x_4}[t^{\alpha 3\kappa\lambda\nu\mu}_{uFAAabdc}(1,2,4,3,5,\dots)-(\alpha\leftrightarrow\nu)]+\sum_{l=5}^{n}\partial_\alpha^{x_l}t^{\alpha l\kappa\lambda\mu\nu}_{uFAAabcd}(1,2,3,4,5,\dots)$$

$$+\tfrac{1}{4}[t^{\kappa 2\lambda\mu\nu}_{uAAAabcd}(1,2,3,4,5,\dots)-(\kappa\leftrightarrow\lambda)]$$

$$+gf_{abr}f_{cdr}\delta(3-4)\tfrac{1}{2}[t^{\mu 2\nu\kappa\lambda}_{uAF}(1,3,2,5\dots)-(\mu\longleftrightarrow\nu)]$$

$$+\frac{g}{2}\{f_{adr}f_{bcr}\delta(2-3)[g^{\mu\kappa}t^{\lambda\nu}_{u\tilde{u}A}(1,3,4,5\dots)-(\kappa\longleftrightarrow\lambda)]\}+\frac{g}{2}\{(c,\mu,3)\longleftrightarrow(d,\nu,4)\}$$

$$+g[f_{acr}f_{dbr}\delta(1-3)-f_{adr}f_{cbr}\delta(1-4)]t^{\mu\nu\kappa\lambda}_{AAF}(3,4,2,5\dots)$$

$$+gf_{abr}f_{cdr}\delta(1-2)t^{\mu\nu\kappa\lambda}_{AAF}(3,4,2,5\dots)], \qquad (A.9)$$

For : $\Omega :=: u^a(x_1)A^b_\mu(x_2)F^c_{\kappa\lambda}(x_3)F^d_{\sigma\tau}(x_4)$:

$$0=\partial_\alpha^{x_1}t^{\alpha\mu\kappa\lambda\sigma\tau}_{AAFFabcd}(1,2,3,4,5,\dots)+\tfrac{1}{2}\partial_\alpha^{x_2}[t^{\alpha 2\mu\kappa\lambda\sigma\tau}_{uAFFabcd}(1,2,3,4,5,\dots)-(\alpha\longleftrightarrow\mu)]$$

$$+\tfrac{1}{2}\{\partial^\lambda_{x_3}t^{\mu\kappa\tau\rho}_{uA\tilde{u}Fabcd}(1,2,3,4,5,\dots)-(\kappa\longleftrightarrow\lambda)]\}$$

$$+\tfrac{1}{2}\{(c,\kappa,\lambda,x_3)\longleftrightarrow(d,\sigma,\tau,x_4)\}+\sum_{l=5}^{n}\partial_\alpha^{x_l}t^{\alpha l\mu\kappa\lambda\sigma\tau}_{uAFFabcd}(1,2,3,4,5,\dots)$$

$$+\tfrac{1}{4}[t^{\kappa 3\mu\lambda\sigma\tau}_{uAAFabcd}(1,2,3,4,5,\ldots)-(\kappa\leftrightarrow\lambda)]+\tfrac{1}{4}[(c,\kappa,\lambda,x_3)\longleftrightarrow(d,\sigma,\tau,x_4)]$$
$$+g\{f_{abr}f_{cdr}\delta(1-2)t^{\mu\kappa\lambda\sigma\tau}_{AAF}(2,3,4,5\ldots)$$
$$-\frac{g}{2}\{f_{adr}f_{bcr}\delta(2-3)[g^{\mu\kappa}t^{\lambda\sigma\tau}_{u\tilde{u}F}(1,3,4,5\ldots)-(\kappa\longleftrightarrow\lambda)]\}$$
$$-\frac{g}{2}\{(c,\kappa,\lambda,x_3)\longleftrightarrow(d,\sigma,\tau,x_4)\}$$
$$+g[f_{adr}f_{bcr}\delta(1-4)-f_{acr}f_{bdr}\delta(1-3)]t^{\mu\kappa\lambda\sigma\tau}_{AFF}(2,3,4,5\ldots), \tag{A.10}$$

For : $\Omega :=: u^a(x_1)F^b_{\mu\nu}(x_2)F^c_{\kappa\lambda}(x_3)F^d_{\sigma\tau}(x_4)$:

$$0=\partial^{x_1}_\alpha t^{\alpha\mu\nu\kappa\lambda\sigma\tau}_{AFFFabcd}(1,2,3,4,5,\ldots)+\tfrac{1}{2}[\partial^\nu_{x_2}t^{\mu\kappa\lambda\sigma\tau}_{u\tilde{u}FFabcd}(1,2,3,4,5,\ldots)-(\mu\longleftrightarrow\nu)]$$
$$+\tfrac{1}{2}[\partial^\lambda_{x_3}t^{\kappa\mu\nu\sigma\tau}_{u\tilde{u}FFacbd}(1,3,2,4,5,\ldots)-(\kappa\longleftrightarrow\lambda)]$$
$$+\tfrac{1}{2}[\partial^\tau_{x_4}t^{\sigma\mu\nu\kappa\lambda}_{u\tilde{u}FFadbc}(1,4,2,3,5,\ldots)-(\sigma\leftrightarrow\tau)]+\sum_{l=5}^{n}\partial^{x_l}_\alpha t^{\alpha l\mu\nu\kappa\lambda\sigma\tau}_{uFFFabcd}(1,2,3,4,5,\ldots)$$
$$+\tfrac{1}{4}[t^{\mu 2\nu\kappa\lambda\sigma\tau}_{uAFFabcd}(1,2,3,4,5,\ldots)-(\mu\leftrightarrow\nu)]+\tfrac{1}{4}[t^{\kappa 2\lambda\mu\nu\sigma\tau}_{uAFFacbd}(1,3,2,4,5,\ldots)-(\kappa\leftrightarrow\lambda)]$$
$$+\tfrac{1}{4}[t^{\sigma 2\tau\mu\nu\kappa\lambda}_{uAFFadbc}(1,4,2,3,5,\ldots)-(\sigma\leftrightarrow\tau)]$$
$$-g[f_{acr}f_{bdr}\delta(1-3)+f_{adr}f_{cbr}\delta(1-4)+f_{abr}f_{dcr}\delta(1-2)]t^{\mu\nu\kappa\lambda\sigma\tau}_{FFF}(2,3,4,5\ldots), \tag{A.11}$$

For : $\Omega :=: u^a(x_1)u^b(x_2)\partial_\mu\tilde{u}^c(x_3)F^d_{\lambda\kappa}(x_4)$:

$$0=\Big[\partial^{x_1}_\alpha t^{\alpha\mu\lambda\kappa}_{Au\tilde{u}Fabcd}(1,2,3,4,5,\ldots)-\Big((a,x_1)\longleftrightarrow(b,x_2)\Big)\Big]$$
$$+\partial^\mu_{x_3}\bar{t}^{3\lambda\kappa}_{uu\tilde{u}Fabcd}(1,2,3,4,5,\ldots)+\tfrac{1}{2}\Big[\partial^\kappa_{x_4}t^{\mu\lambda}_{uu\tilde{u}\tilde{u}abcd}(1,2,3,4,5,\ldots)-(\kappa\leftrightarrow\lambda)\Big]$$
$$+\sum_{l=5}^{n}\partial^{x_l}_\alpha t^{\alpha l\mu\lambda\kappa}_{uu\tilde{u}Fabcd}(1,2,3,4,5,\ldots)$$
$$+\tfrac{1}{4}[t^{\lambda 4\mu\kappa}_{uu\tilde{u}Aabcd}(1,2,3,4,5,\ldots)-(\lambda\leftrightarrow\kappa)]$$
$$+gf_{abr}f_{cdr}\delta(1-2)t^{\mu\lambda\kappa}_{u\tilde{u}F}(2,3,4,5\ldots)$$
$$-g[f_{acr}f_{bdr}\delta(1-3)t^{\mu\lambda\kappa}_{u\tilde{u}F}(2,3,4,5\ldots)-((a,x_1)\longleftrightarrow(b,x_2))]$$
$$+g[f_{adr}f_{bcr}\delta(1-4)t^{\mu\lambda\kappa}_{u\tilde{u}F}(2,3,4,5\ldots)-((a,x_1)\longleftrightarrow(b,x_2))], \tag{A.12}$$

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